Some message passing results from SP4

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in collaboration with

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Outline

- Message passing as minimization procedures

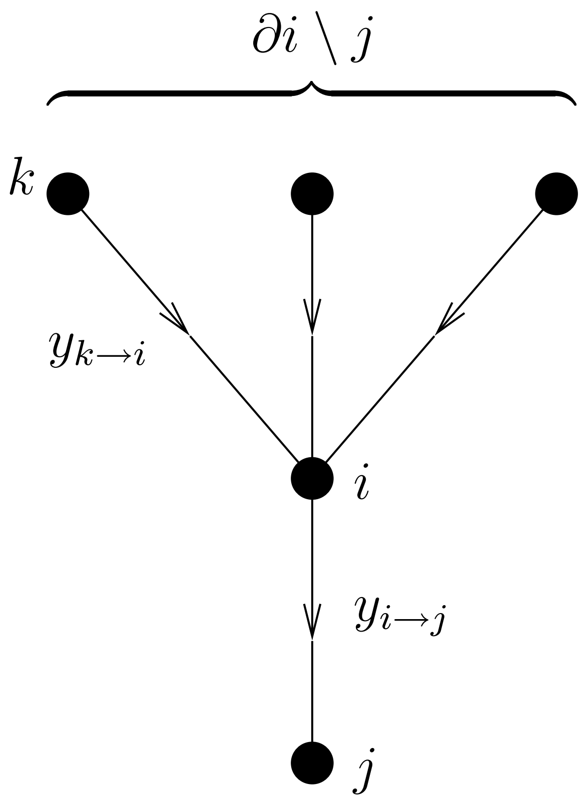
  Application : approximate counting of exponentially numerous objects in polynomial time

  [with E. Marinari and R. Monasson]

- Other message passing algorithms

  Application : measuring barriers

  [with A. Montanari]



$$y_{i\to j} = g(\{y_{k\to i}\})$$

Belief propagation

Configuration space $\underline{S} = \{S_1, \dots, S_N\}$

Weight function $w(\underline{S}) \geq 0$

Probability law $p(\underline{S}) = w(\underline{S})/Z, \qquad Z = \sum_{\underline{S}} w(\underline{S})$

Variational formulation :

$$\ln Z = -\min_{p_{\rm v}} \sum_{\underline{S}} p_{\rm v}(\underline{S}) \ln \left(\frac{p_{\rm v}(\underline{S})}{w(\underline{S})} \right)$$

Minimization of the Bethe approximation of this functional
$\Rightarrow$ Belief propagation

[Survey propagation can be recast under this form]

Counting with Belief Propagation

If $\mathcal{A}$ is an interesting subset of the configuration space, $\mathcal{N}$ its cardinality

$$\text{Define } \; w(\underline{S}) = \begin{cases} 1 & \text{if } \; \underline{S} \in \mathcal{A} \\ 0 & \text{otherwise} \end{cases} \qquad \Rightarrow \qquad Z = \mathcal{N}$$

Devise the corresponding graphical model, and its Bethe approximation
$\Rightarrow$ a Belief Propagation algorithm for counting

Interesting when $\mathcal{N} = e^{N\sigma}$, exhaustive enumeration impossible

Counting loops with Belief Propagation

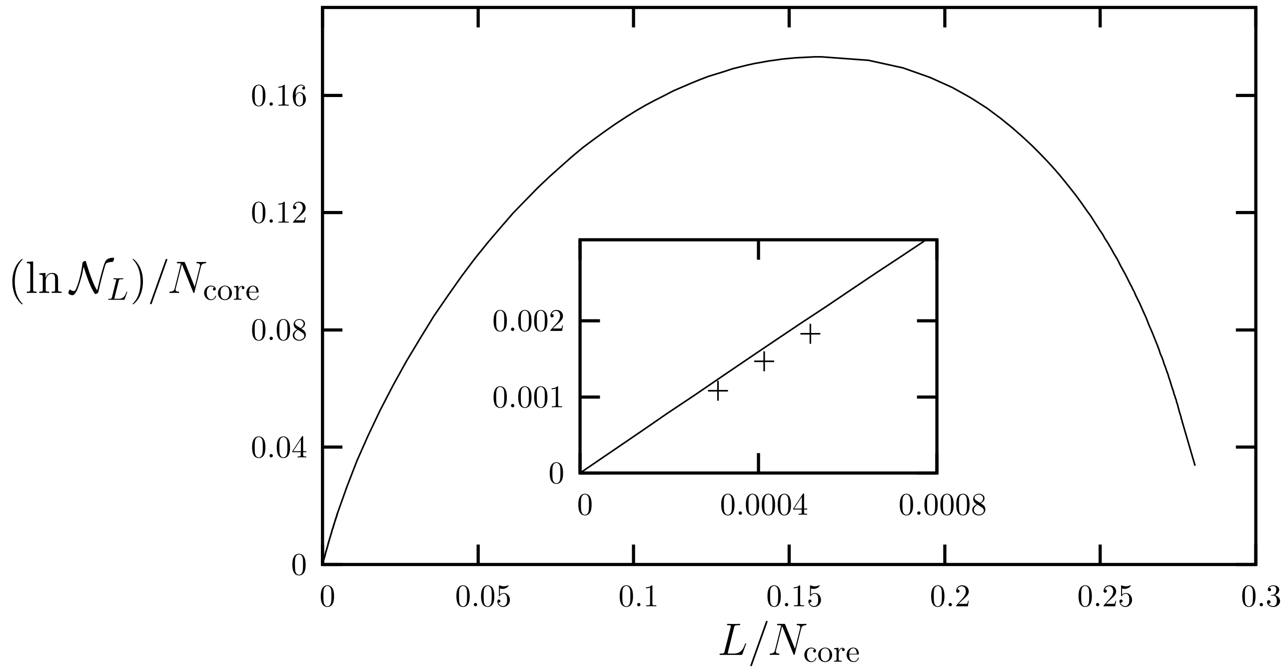
Motivations :

- Large amount of experimental datas on real-world networks (Internet for instance), characteristic features to (in)validate proposed models ?
- Local properties (connectivity distribution, clustering coefficient, number of short loops,...) easy to measure.
- Global properties (long loops or other large patterns) difficult :
  - exponential number of long loops $\Rightarrow$ tricky to enumerate
  - deciding if there is an Hamiltonian circuit is NP-complete

DIMES data for the Internet structure

$N = 14921$

$M = 33666$



$\approx 10^{729}$ loops

most numerous have ≈ 1555 edges

longest have ≈ 2710 edges

for $L = 3, 4, 5$, compares reasonably with exact enumeration (inset)

Typical numbers of loops in random graphs

« Confirmation » of mathematical conjectures and new quantitative ones

Perspectives

- Belief-inspired decimation algorithm to construct cycles
- Alternative measure of centrality
- Other random graph models, correlated networks
- Rigorous results from this approach
- Beyond Bethe approximation [Montanari and Rizzo]

Other message passing algorithms

Can be used a priori for any quantity which has a recursive construction for rooted trees

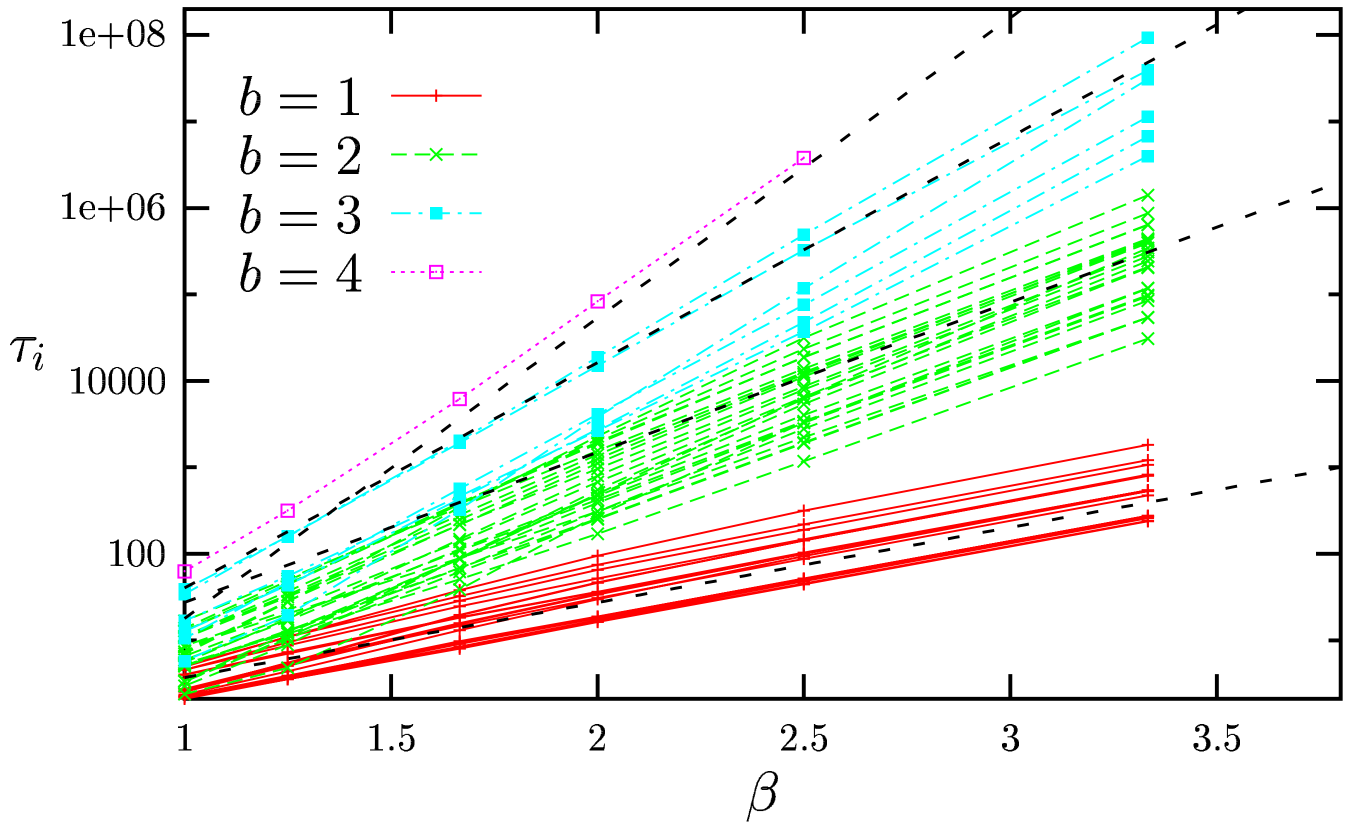
Example : energy $E(\underline{S})$, trajectories $\underline{S}(t)$ with local moves

$$\text{Barrier :} \quad b = \min_{\underline{S}} \max_{t} E(\underline{S}(t)) \tag{1}$$

With detailed balance at (small) temperature T, Arrhenius law $\tau \sim \exp[b/T]$

For the simplest model (p-spin, XORSAT), can be done (with complicated messages)

Comparison with local correlation times obtained through Monte Carlo simulations :



Perspectives

- application to kinetically constrained models (related to core percolation)

- other optimization problems (satisfiability, coloring)

- helpful to understand the behaviour of local search algorithm (without detailed balance), surprisingly good on random problems